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COORDINATE SYSTEMS FOR DIFFERENTIAL CORRECTION

by E. A. Emerson and W. W. Lemmon

Prepared by
TRW SYSTEMS
Redondo Beach, Calif.
for Langley Research Center

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ABSTRACT

This paper presents a system of state transition partial derivatives developed at TRW Systems for which the tracking information normal matrix for a lunar orbiter is nearly diagonalized. A simulated tracking study shows the well-conditioning of the normal matrix for a lunar orbiter over one lunar revolution.

LIST OF SYMBOLS

A	Matrix of partial derivatives of observations with respect to regression variables
$C_{m,n}$ $S_{m,n}$	Coefficients of the spherical surface harmonics in the expansion for the moon's gravitational potential
I	Identity matrix
M	Estimate of normal matrix elements from continuous approximation
N	Estimated "per unit time" normal matrix
O	Vector observations
R	Continuous analog to A matrix
S	Coefficients of Fourier decomposition of R
T	Total interval of tracking
T_o	Epoch at which osculating orbital elements are defined
U	Coordinate transformation matrix
U_o	Initial conditions for integrating variational equations
X	State vector of system, including osculating orbital elements and other parameters
Y	Vector of residuals
Z	State vector in coordinate system in which differential correction is calculated
f,g	Arbitrary coefficients
r	Orbit plane coordinates of vehicle position
v	Orbit plane coordinates of vehicle velocity
x	Element of state vector
z	Element of state vector
Ω	Moon's rotation rate
α	Elements of differential vector in which differential correction is calculated

LIST OF SYMBOLS (continued)

ϵ Unit normal to plane of vehicle motion

η Continuous residual function

μ GM (gravitational constant multiplied by mass of the moon).

ν White noise (in statistical formula), vehicle revolution rate (in Fourier decomposition of R)

ϕ Arbitrary coefficient

ω Orbital energy parameter

COORDINATE SYSTEMS FOR DIFFERENTIAL CORRECTION

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TRW Systems

1. SUMMARY

This report presents a system of state transition partial derivatives, developed at TRW Systems, for which the tracking information normal matrix for a lunar orbiter is nearly diagonalized. This result is obtained by using a set of regression variables (α variables) which, for an earth-centered tracker and assuming circular orbits for moon and satellite, precisely diagonalize the normal matrix. An analytic estimate of the correlations between the variables (for typical tracking data rates over one revolution of the moon about the earth) indicates that their maximum correlation approaches $\sqrt{2/3}$ as the limiting case. The analytical results were demonstrated in a simulated tracking study.

This report does not present the theoretical basis for these partial derivatives, nor the method of their derivation. The complete derivation and a history of the concept will be available in Reference 1.

2. INTRODUCTION

A persistent problem associated with statistical orbit determination programs is that the tracking information normal matrix can become ill-conditioned with time. This matrix is used in the calculation of the differential correction for improving the estimate of orbital elements and other parameters, and its inverse can under certain conditions indicate the standard errors in the corrections. When a matrix is sufficiently ill-conditioned, it can cause inaccurate corrections and a meaningful inverse may be impossible to obtain. This report presents the results of a continuing study of one approach to solving this problem, but it does not contain the body of theory on which the techniques and derivations are based. The original ideas behind the work represented by the results in this report (the limiting values of correlations for a lunar orbiter) are those of Mr. W. W. Lemmon alone. The basic theory has recently been documented (References 1 and 2), but the detailed derivations of all of Mr. Lemmon's work are not available. The results are presented here with only brief explanations. It is assumed that the reader is already generally familiar with the nature and sources of singularities in the tracking information normal matrix, the state transition matrix as the solution of the variational equations, the basic method of special perturbations when applied to orbit determination, and the basic method of iterated linearly determined differential corrections as the solution to the nonlinear statistical orbit determination problem.

The technology derived from this approach has been under development at TRW Systems for some two years; the results of this development can be utilized quite simply. At TRW Systems, the method has been incorporated in a series of computer programs for tracking error analysis of both earth and lunar orbiters. The expected improvement in performance has been found with these programs.

The method used depends on establishing special independent variables with respect to which the state transition partial derivatives are calculated. In most existing programs for statistical orbit determination, these variables are already some fairly conventional set of osculating orbital elements*. The independent variables, however, need not be any easily interpreted set of elements. It is desirable from the computation point of view that they have certain properties, such as being a linear transformation of the osculating elements in which the equations of motion are integrated, and being formulated in such a way that inversion of the state transition matrix is trivial. Otherwise, they are free to be as bizarre as necessary, up to the point where they lose any sensible physical interpretation.

This report covers what was to be an investigation of a choice of orbital elements to be used as the independent variables. Since the method presented here leads to those variables which are optimum; i.e., which diagonalize the normal matrix for a reasonable model of the tracking and orbital geometry, it was found to be of little or no value to examine other possible choices of elements.

Basic to the present method is the concept of establishing variables which separate energy and orientation (direction of displacement) rather than position and velocity. The method also uses variables which are not degenerate under commonly-occurring conditions such as zero eccentricity and zero inclination. Variables possessing these characteristics to varying degrees have been called "α-variables" in the literature, and this term has been used in particular to designate a set of variables developed at Goddard Space Flight Center which separate energy and orientation by the use of the Hamiltonian and Lagrangian functions.

This general method is referred to in this report as an "alpha system", and the contribution presented here consists first of a transformation from the cartesian elements to a set of α-variables which will diagonalize the normal matrix of a lunar orbiter, and second of an analytic estimate of maximum correlations to be expected when solving for coefficients of the lunar gravitational potential model.

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*The term "orbital element" means here any set of six parameters (and a time) which define (with reference to a given coordinate system) the position and velocity of a satellite in a Keplerian orbit.

3. STATEMENT OF THE PROBLEM

In a computer program for statistical orbit determination from tracking data, an ill-conditioned normal matrix will give rise to problems in connection with the computational process which calculates the least-squares solution to a linearized differential correction process by solving a system of equations represented in their simplest form by

$$\delta X = (A^T A)^{-1} A^T \delta Y$$

δX is the differential correction applied to an n dimensional state vector X_o.

X is the state vector of the system. X typically contains elements which specify the position and velocity of a spacecraft, but may be extended or substituted to contain physical constants specific to the mission (spacecraft effective radiation pressure cross section), physical constants of nature (gravitational model coefficients), physical constants of the tracking system (biases, systematic errors), and spacecraft originated perturbations (thrusts, attitude control system perturbations).

For practical purposes, the dimension of X is limited to the dimension of the matrix which the tracking program can accommodate. The analysis in this paper is concerned with X containing the following parameters:

X_o	Cartesian position and velocity elements
μ	The lunar gravitational constant
$C_{2,0}$ $C_{3,0}$ $C_{4,0}$	Zonal coefficients of the lunar potential field
$C_{2,1}$ $C_{3,1}$ $S_{2,1}$ $S_{3,1}$	Tesseral harmonic coefficients of the lunar potential field
$C_{2,2}$ $S_{2,2}$	Sectorial harmonic coefficients of the lunar potential field

X_o is the state vector at epoch T_o. The position and velocity elements of X_o are the initial conditions for the numerical integration of the equations of motion of the spacecraft.

A^TA is the n x n tracking information matrix, the "influence matrix"

A is the m x n matrix of partial derivatives relating m particular observational residuals at known times to the state vector at (usually) some other epoch time, T_O. An element of A has the form $\partial O_t/\partial x_{jT_O}$; that is, the partial derivative of the observation O at time t with respect to some element x_j in the solution vector at time T_O. This partial derivative is the sum over i of the separate factored partial derivatives $(\partial O_t/\partial x_{it}) \cdot (\partial x_{it}/\partial x_{jT_O})$ where $\partial O_t/\partial x_{it}$ is the geometric partial derivative of the observation at time t with respect to the i^{th} element of the solution vector at time t, and $\partial x_{it}/\partial x_{jT_O}$ is the element of the state transition matrix relating the variation of the i^{th} element at time t, x_{it}, to the variation of the j^{th} element at time T_O, x_{jT_O}.

δY is the m x 1 matrix of residuals calculated by taking the difference between actual observations and the observations which should have resulted had the spacecraft been in the state calculated by integrating the equations of motion.

Two most important results are derived from solving this system of equations. The first is the differential correction δX to the state vector. The validity of this correction depends on the validity of the assumptions of linearity and the well-conditioning of the numerical process. The second is the inverse normal matrix $(A^TA)^{-1}$ which, under assumptions including linearity, is identified with the covariance matrix indicating the statistics on the solution vector δX. The correctness of the elements of the covariance matrix, aside from errors in assumptions, depends on the well-conditioning of the numerical process.

The numerical process can become ill-conditioned in two ways:

1) The normal matrix A^TA may be nearly singular because of very high correlation between two or more of the elements of X. Such high correlation may be due to the fact that two elements are jointly correlated with single cause, or can result from some fortuitous short-term coincidence of orbit characteristics and sensor geometry which makes some elements either indeterminate or redundant for the data set under consideration.

2) The computational process has limited precision, and it would be possible for the matrix operations to take the difference of two nearly equal numbers and lose all but a very few significant digits. This can occur almost anywhere in the compu-

tation, but is more likely in the calculation of $(A^TA)^{-1}$. It is not always possible to disregard it in the calculation of δX, either.

Other problems closely related to the problem of ill-conditioning are:

- Failure of the assumption of linearity
- Inadequate word length for accumulation of the A^TA matrix
- The dimension of the solution vector in relation to the calculating precision required
- Insufficient accuracy in calculation of the partial derivatives.

These problems are not considered in the present study, and the remedy suggested for the ill-conditioning problem is not applicable to them.

The ill-conditioning problem can be treated by correcting either or both of the two contributory causes listed above. With respect to the first, it is desirable to use a solution vector whose elements are uncorrelated, or more exactly to calculate partial derivatives with respect to a set of variables which are uncorrelated (and which may subsequently be rotated in n-space to the required elements). This set of variables may in practice be established in one of two ways:

- They may be calculated in advance on the basis of theoretical considerations (i.e., orbit and tracking geometry)
- They may be obtained pragmatically by calculating for an obtained A^TA matrix that rotation which, if applied to the initial elements, would have made the matrix diagonal. This rotation is then applied to subsequent iterations.

This report gives the results of using the former method; that is, it gives a set of variables designed on the basis of tracking and orbit considerations to cause the resulting A^TA matrix to be well-conditioned.

With regard to the second cause of matrix inaccuracy, it is possible to extend the precision with which various matrix-related calculations are carried out. Extending precision is simple in concept and not difficult from the programming point of view; it has been done in portions of the JPL Orbit Determination Program (ODP).

4. THE ALPHA SYSTEM APPROACH TO DIFFERENTIAL CORRECTION

The technique of the alpha system involves defining the state vector X_o, which represents the initial conditions for integrating the equations of motion, and the correction δX to be applied to that state vector. The equations of motion are usually integrated in a coordinate system in which the formulation is simple and symmetrical (Cartesian) or in which the computation time may be systematically minimized (variation of classical orbital parameters) or in which some geometric factor is predominant (topocentric). The initial conditions must be provided in this coordinate system, and if the differential correction is iterated, the correction must in some way ultimately be applied in this coordinate system.

In the JPL Orbit Determination Program, the initial conditions X_o and the independent variables of the A matrix are the same cartesian elements. The calculated correction X is applied directly, so that $X_1 = X_o + \delta X$. In the TRW AT85 program (earth satellite version), the initial conditions X_o are cartesian elements and the independent variables Z_o of the A matrix are conventional polar spherical elements. It is necessary at the beginning of every iteration to know both X_o (in order to initialize the equations of motion) and Z_o (to permit the correction to be applied; $Z_1 = Z_o + \delta Z$). X_1 is obtained from Z_1 by a linear transformation $X_1 = UZ_1$. Both of the above approaches may be summarized in the following way:

Initial value of the state vector in the correction coordinate system	Z_o
Initial conditions for the equations of motion, where U is the required transformation. If the equations of motion are integrated in the Z_o system, U = I, the identity matrix	$X_o = U Z_o$
Correction to Z_o, where a typical element is of the form O_t/z_{iT_o}	$Z_o = (A^TA)^{-1}A^T \delta y$
Corrected Z	$Z_1 = Z_o + Z_o$
Corrected X	$X_1 = U Z_1$

In the alpha-system, the approach is slightly different; no interpretation is given to the correction coordinate system, which is the alpha-system. It is simply a "differential" coordinate system in which the normal matrix $A^T A$ is well-conditioned. The derived correction $\delta\alpha$ is rotated to make it a correction δX_o in the coordinate system of the initial conditions and is then applied to complete the differential correction process. The partial derivatives $\partial O_t / \partial \alpha_{iT_o}$ which constitute the A matrix are obtained by integrating the Cartesian variational equations with particular initial conditions $\partial X_o / \partial \alpha_{T_o}$. It is the correct specification of this initial condition matrix which produces the desired α-system. The specification of an α-system is the specification of this initializing matrix. The α-system process may be summarized as follows:

Initial value of the state vector in the coordinate system used for the equation of motion $\qquad X_o$

Initial conditions for the variational equations where f is the function specifying the α-system

$$U = \partial X_{T_o} / \partial \alpha_{T_o} = f\,(X_{T_o})$$

Correction in the α-system, where a typical element of A is of the form $\partial O_t / \partial \alpha_{iT_o}$

$$\delta\alpha_{T_o} = (A^T A)^{-1} A^T\, \delta y$$

Correction in coordinate system for equations of motion

$$\delta X_o = U\, \delta\alpha_{T_o}$$

Corrected X

$$X_1 = X_o + \delta X_o$$

The design and choice of U depends on the tracking geometry, the types of observations, the duration of tracking, the evolution of the orbit, and the components of the solution vector over and above the position and velocity. Having alternate choices for U permits tailoring the α variables to fit the specific problem; a recommended set for the lunar orbiter is given below. U is specified by establishing a function of X_o that considers the design factors. Once the function is derived and established, it is made a part of the computation and is used to calculate, as a function of the original and subsequently corrected X_o, the initial conditions for the variational equations. The required subsequent partial derivatives $\partial O_t/\partial \alpha_{iT_o}$ are then developed.

The method of designing the U function is based on the general principle of separating energy from orientation, together with the requirement to eliminate degeneracy. The theory of the method is strongly dependent on a new formulation for conic trajectory relations (Reference 2). Obtaining any particular U function requires an understanding of the influence of the statistical behavior of residuals on the differential correction as well as a knowledge of the influence of system dynamics and motion on the computation of partial derivatives.

The difficulty of obtaining the U function can be appreciated from a consideration of the number of variables must be taken into account and the many approximations required. For uncomplicated cases (e.g., standard position fix observations, no very extended solution vectors, moderate orbit evolution, no powered flight) some standard α-systems are available and are tailored to operate satisfactorily if certain orbital eccentricities are avoided. For conventional orbiting satellites, for example, the so-called "parabolic degenerate" system is adequate, while for planetary escape trajectories the "circular degenerate" system is sufficient. No method for designing U functions for general applications has been formulated, and their design thus retains certain aspects of an art rather than a science.

The α-system recommended for the lunar orbiter is derived from the parabolic degenerate system; that is, it has degeneracies only for eccentricities very nearly equal to unity. It includes the coefficients of the gravitational potential harmonics and includes the mass of the moon in the form of μ. The assumptions on which this α-system are based are as follows:

- A single tracking station is located at the center of the earth and takes range and range rate data at a rate that is high in relation to the period of the satellite. Data is provided over a period of one revolution of the moon about the earth.
- Tracking errors are stationary and normally distributed.

- The satellite is revolving about the moon taken as a point mass and is in a circular orbit moderately inclined to the lunar equator.
- The moon is in a circular orbit about the earth.
- No account is taken of perturbations of the vehicle's motion caused by the earth and sun.

For this model, the 6 x 6 normal matrix for the primitive parabolic degenerate α-system is diagonalized. In nature, this model is only an approximation, and it is expected that a real normal matrix for the primitive system would not be diagonalized, but that minimally it would be very well-conditioned. This expectation has been borne out in simulated runs for the lunar orbiter. The test of well-conditioning applied to the simulation was a comparison of the normal matrix with the inverse of its inverse.

Recall that the only requirement to institute an α-system is to provide initial conditions for variational equations. This is because of the chain of contributing partial derivatives and because the superposition theorem applies to the solution of the linear variational equations.

$$\frac{\partial O_t}{\partial \alpha_{iT_o}} = \sum_j \left(\sum_k \frac{\partial X_{kT_o}}{\partial \alpha_{iT_o}} \cdot \frac{\partial X_{jt}}{\partial X_{kT_o}} \right) \cdot \left(\frac{\partial O_t}{\partial X_{jt}} \right)$$

$\dfrac{\partial O_t}{\partial X_{jt}}$ is an analytic expression.

$\dfrac{\partial X_{jt}}{\partial X_{kT_o}}$ is an element from the solution of the variational equations with I as the starting conditions.

$\dfrac{\partial X_{kT_o}}{\partial \alpha_{iT_o}}$ is an element from the U matrix which defines the α-system.

Obtaining $\dfrac{\partial X_{kT_o}}{\partial \alpha_{iT_o}} \cdot \dfrac{\partial X_{jt}}{\partial X_{kT_o}} = \dfrac{\partial X_{jt}}{\partial \alpha_{iT_o}}$ is computationally equivalent to using U as the initial conditions of the variational equations. Thus, by using U instead of I as initial conditions to the variational equations, the solution is directly $\partial X_t / \partial \alpha_{jT_0}$ for correctly selected α-variables may be inverted by rearrangement and sign changes alone. (See reference 3.)

5. AN ALPHA-SYSTEM FOR THE LUNAR ORBITER

Given that the principal coordinate system of integration is Cartesian, referenced to the spacecraft orbit plane and the direction from the force center to vehicle position at epoch, T_0, the U matrix may be defined. The following notation is used:

$$\vec{r}_o = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix}, \quad \text{position at } T_o$$

$$\vec{V}_o = \begin{bmatrix} \dot{X}_o \\ \dot{Y}_o \\ \dot{Z}_o \end{bmatrix}, \quad \text{velocity at } T_o$$

μ = Gravitational constant

$\omega = 2\mu/r_o - v_o^2$ = negative twice the orbital energy

$\vec{\epsilon}$ = Unit normal to plane of vehicle motion

T = Interval of tracking

α_1	= Column 1 of U_o	(6×1)
$\alpha_{\bar{1}}$	= Column 2 of U_o	(6×1)
α_2	= Column 3 of U_o	(6×1)
$\alpha_{\bar{2}}$	= Column 4 of U_o	(6×1)
α_3	= Column 5 of U_o	(6×1)
$\alpha_{\bar{3}}$	= Column 6 of U_o	(6×1)
α_μ	= Column 7 of U_o	(6×1)
α_j	= Columns 8-16 of U_o	(6×9)
U	$= \alpha_1 \; \alpha_{\bar{1}} \; \alpha_2 \; \alpha_{\bar{2}} \; \alpha_3 \; \alpha_{\bar{3}} \; \alpha_\mu \; \alpha_j$	(6×16)

To construct a "primitive" U_o, the following definitions are used:

$$\alpha_1 = \begin{bmatrix} \vec{V}_o \\ -\mu\vec{r}_o/r_o^3 \end{bmatrix}$$

$$\alpha_{\bar{1}} = \begin{bmatrix} 2\vec{r}_o/\omega + 3T\vec{V}_o/2\omega \\ \vec{V}_o/\omega - 3T\mu\vec{r}_o/2\omega r_o^3 \end{bmatrix}$$

$$\alpha_2 = \begin{bmatrix} \vec{\epsilon} \times \vec{V}_o \\ \mu\vec{\epsilon} \times \vec{r}_o/r_o^3 \end{bmatrix}$$

$$\alpha_{\bar{2}} = \begin{bmatrix} -2\vec{\epsilon} \times \vec{r}_o/\omega \\ \vec{\epsilon} \times \vec{V}_o/\omega \end{bmatrix}$$

$$\alpha_3 = \begin{bmatrix} \vec{\epsilon} \\ 0 \end{bmatrix}$$

$$\alpha_{\bar{3}} = \begin{bmatrix} 0 \\ \vec{\epsilon} \end{bmatrix}$$

$$\alpha_\mu = (r_o V_o^2 + \mu)\,\alpha_{\bar{1}}/3r_o + |\vec{r}_o \times \vec{V}_o|\,\alpha_1/\mu\omega$$

$$X_j = 0$$

It was stated above that the α-system does not correspond to variations in any conventional element set. The partial derivatives do relate the observational residuals to some othogonal variation parameters in phase space. These parameters do not have any physical interpretation, but the following quasi-physical interpretations are offered without justification:

α_1	Variation in epoch
$\alpha_{\bar{1}}$	Variation in orbital energy
$\alpha_2, \alpha_{\bar{2}}$	A conjugate pair that define variation in an "eccentricity" and variation of apocentron
α_3	Variation in out-of-plane position
$\alpha_{\bar{3}}$	Variation in out-of-plane velocity
α_μ	Variation in period
α_j	Variation in gravitational potential coefficients

These uncommon parameters suggest other more familiar components; by way of a second-order, less precise interpretation they can be considered to correspond roughly to the following conventional items:

α_1	Variation in reciprocal mean motion
$\alpha_{\bar{1}}$	Variation in energy
α_2	Variation in eccentricity
$\alpha_{\bar{2}}$	Variation in argument of apocentron
α_3	Variation in displacement from orbit plane
$\alpha_{\bar{3}}$	Variation in velocity normal to orbit plane

Notwithstanding these rather vague "interpretations", the corrections $\delta\alpha$, once calculated by solving the system of normal equations, may be rotated directly back to the original coordinates by the relation

$$\delta X_o = \frac{\partial X_o}{\partial \alpha} \delta\alpha = U \, \delta \alpha_o$$

without ever interpreting the α-variables. Their only requirement is that the matrix U must be convenient to invert and that the $A^T A$ matrix be well-conditioned.

6. THE ANALYTICALLY DERIVED NORMAL MATRIX N

It was not anticipated that the contract covering the work reported here would provide for modification of a computer program to permit testing an α-system for tracking a lunar orbiter. The α-system was evaluated by an analytic study to estimate the limiting values of the elements of the normal matrix.

To a certain extent the evaluation is circular because some of the assumptions necessary to the averaging of the tracking data were already applied in the selection of the α-system. Otherwise, the analytical estimate introduces approximations based on statistical and mathematical assumptions. It can be shown, however (although it is not shown here), that any errors introduced have a mean value of zero. It is worthy of note that the α-system is a system of computing partial derivatives which is orderly and elegant enough that an analytic estimate of the normal matrix can indeed be obtained. The more important statistical and mathematical assumptions permitting estimation of the normal matrix are given below; they will clarify the symbols used in evaluating the elements in Figure 1. The entries given in Figure 1 are the limiting values of the "per unit time" normal matrix $N = \frac{1}{T}(A^TA)$ under the assumptions given.

6.1 CONTINUITY APPROXIMATION

Tracking is assumed to be continuous and a continuous, time-dependent observation residual vector function $\delta\eta$ is obtained as follows:

$$\delta\eta = R\,\delta\xi + \delta\nu$$

where $\delta\xi$ is the continuous time dependent spacecraft state vector variation function. $\delta\xi$ is the continuous analog of δy.

$R = \partial\eta/\partial\xi$, a continuous function of time
(R is the continuous analog of A)

$\delta\nu$ = white noise

The A^TA matrix is a tracking accuracy normal matrix summed discretely over an interval of time, T, from the weighted partial derivatives at those

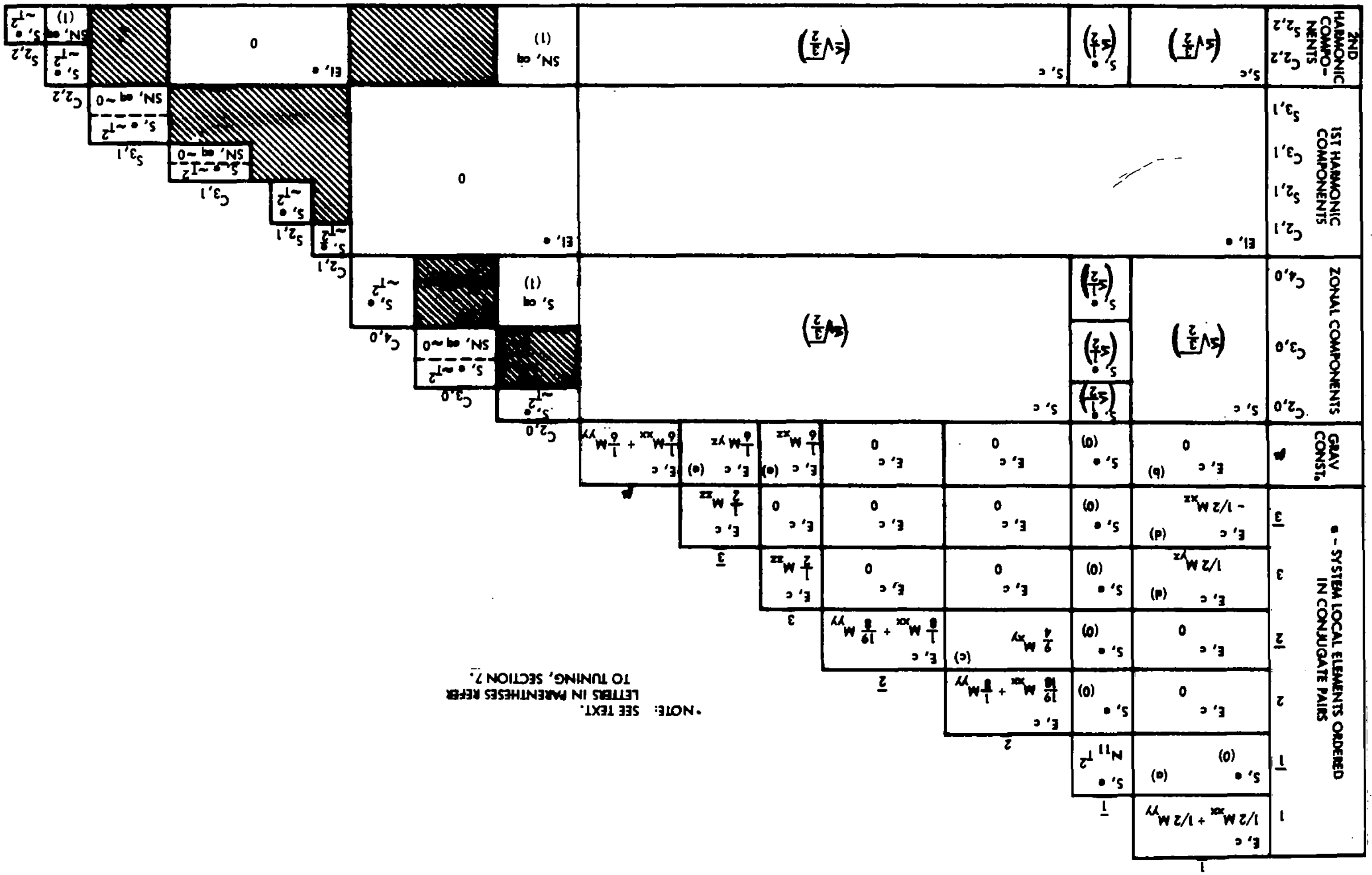

Figure 1.-Analytically Derived Normal Matrix N

times when observations occur. The interval of time is arbitrary, and in the analytic approximation it has been averaged out. The resulting normal matrix as derived is then the "per unit time" normal matrix N.

$$N = \frac{1}{T}(A^TA)$$

The A^TA matrix arises in computation only as a necessity to solve a system of normal equations to estimate a correction vector. The substitution of a continuous process for a discrete process requires an integration in lieu of a summation. The continuous process requires by analogy the evaluation of the integral

$$M = \frac{1}{T}\int_{t_o}^{t_o+T} R^TR \, dt$$

where t_o and $t_o + T$ are the beginning and end of the tracking interval. M is by construction positive, semi-definite, and symmetric. For the analytically estimated normal matrix, referenced to the vehicle-centered spacecraft orbit plane coordinates, M is referenced to the orbit plane and to the line from the force center to the vehicle at epoch.

6.2 TRACKING MODEL

Statistical averages over the interval T were dependent upon assumptions of integrability, stationarity, and an ergodic model. The integrability requirement is obvious. The stationarity assumption is very weak, and was the only assumption used in entries coded with an "S". The only stationary assumption is that the sum of the components $M_{XX} + M_{YY}$ of the M matrix is a function bounded and Fourier transformable for all T and converges uniformly to a nonzero value as T approaches infinity. The ergodic model requires that R can be decomposed in the Fourier form

$$R = R_e \left[S_o \, e^{i\Omega t} + \sum_{j=1}^{\infty} S_j \, e^{i\nu_j t} \right]$$

where Ω is the moon's rotation rate, and the ν_j's are multiples of the vehicle's orbital rate; and that

$$\Omega \pm \nu_j \neq 0, \quad \text{all } j$$

$$\Omega \pm \nu_j \neq \nu_l, \quad \text{all } j$$

$$\nu_m \pm \nu_n \neq \nu_l, \quad \text{all } m \neq n$$

That is, there are no resonances.

Statistical tracking models are designated in Figure 1 as follows:

Key	
S	Stationary
SN	Stationary with no sensitivity to position and velocity deviations normal to the moon's equator
E	Ergodic (without range rate)
EI	Ergodic tracking for T = an integral number of lunar months (= revolutions)

6.3 ORBIT MODEL

The spacecraft orbit with respect to the moon's center was taken to be one of four types designated by key.

Key		
e	Elliptical inclined	$(e \neq 0, i \neq 0)$
c	Circular inclined	$(e = 0, i \neq 0)$
eq	Elliptical equatorial	$(e \neq 0, i = 0)$
cq	Circular equatorial	$(e = 0, i = 0)$

6.4 INTERPRETATION OF OFF-DIAGONAL ELEMENTS

The principal entry in each element of Figure 1 differs whether the element is on the main diagonal or off the main diagonal. Elements on the main diagonal are given as averaged values over the continuously defined partial derivative matrix R^TR. Elements off the main diagonal are estimates of the "correlation-like" function $N_{ij}(N_{ii}N_{jj})^{1/2}$. These are not correlations (the inverse of N is the theoretical covariance matrix from which statistical correlations would be derived) but are values generally indicative of the well-conditioning of the normal matrix. If any of these values were identically unity, then the matrix would be singular. If all are zero, the matrix is diagonal. The time and distance units for nonzero entries are the period and major semiaxis respectively.

The entries given are zero if in the limit as T approaches infinity the expected "correlation" is zero. Where the "correlation" approaches and is bounded by some numerical value, that value is given in parentheses. (For something which is always zero, zero is given in parentheses). In some entries, the resulting "correlations" depend on characteristics of R. For these cases, it has been found that further "tuning" of the α variable will eliminate these nonzero values off of the diagonal. Tuning information is given in section 7 following.

7. TUNING

The word "tuning" is used to designate the process of taking linear combinations of the columns of the primitive U matrix in such a way as to eliminate nonzero off-diagonal elements in the normal matrix. The process was used in defining the α-system for the lunar orbiter, as outlined below. The letter designations refer to notes indicated on Figure 1, where offending off-diagonal "correlations" appear.

a) Time Phasing. When epoch is not at the center of the data, replacing the primitive $\alpha_{\bar{1}}$ $(= [2\bar{r}_0/\omega,\ -\bar{V}_0/\omega]^T)$ with $\alpha_{\bar{1}} + 3T\alpha_1/2\omega$ decouples α_1 and $\alpha_{\bar{1}}$. This has been done in the recommended U.

b) Gravitational Constant - Energy Decoupling. The element μ is naturally coupled with the energy, but if the α_μ as defined for U replaces μ proper, then the "correlation" is eliminated.

c) Gauging of α_2. The α_2, $\alpha_{\bar{2}}$ "correlation" can be eliminated by reorienting the direction of variation away from apocentron. This requires the following replacements:*

$$f_2\,\alpha_2 + \omega f_{\bar{2}}\,\alpha_{\bar{2}} \longrightarrow \alpha_2$$

$$g_2\,\alpha_2 + g_{\bar{2}}\,\alpha_{\bar{2}}/\omega \longrightarrow \alpha_{\bar{2}}$$

where $$f_2 g_{\bar{2}} - g_2 f_{\bar{2}} = 1$$

d) Decoupling of α_1, α_3. The "correlations" α_1, α_3 and α_1, $\alpha_{\bar{3}}$ can be eliminated by the replacements

$$\alpha_3 + \phi_{13}\,\alpha_1 \longrightarrow \alpha_3$$

$$\alpha_{\bar{1}} - \phi_{13}\,\alpha_{\bar{3}} \longrightarrow \alpha_{\bar{1}}$$

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*The method of choosing the coefficients f, g, and ϕ used in items c, d, e, and f of this section has not been documented. The method presented here is given to show that it is possible to eliminate the off-diagonal entries.

e) Decoupling of α_μ, α_3. The "correlations" α_μ, α_3 and α_μ, $\alpha_{\bar{3}}$ can be eliminated by the replacement

$$\alpha_\mu + \phi_{\mu\bar{3}}\,\alpha_{\bar{3}} \longrightarrow \alpha_\mu$$

f) Gauging α_3. Steps (d) and (e) above could cause α_3, $\alpha_{\bar{3}}$ to be "correlated". This result can be avoided by a device similar to that used in item (c) above.

8. COMPARISON OF α-VARIABLES WITH CARTESIAN VARIABLES

In the TRW TAPP IV (Tracking Accuracy Prediction Program) for application to earth satellite and lunar probe tracking problems, both parabolic degenerate α-variables and Cartesian variables are programmed. Either may be selected. This program also has provision for checking on the well-conditioning of a normal matrix by the following sequence:

- Inverting the matrix in double precision
- Truncating the inverse to single precision
- Inverting the truncated inverse again in double precision
- Printing all three matrices in single precision

The twice inverted matrix can then be compared to the original for agreement. Although this test has certain disadvantages, it is if anything too strict.

The TAPP program models the motion of the spacecraft with a two-body Kepler orbit about the force center. In the lunar orbiter case the force center is the moon, whose ephemeris is derived from a fairly simple analytic model. The periods during which tracking data is provided were selected to satisfy the requirements for visibility of the spacecraft from the Goldstone, Madrid, and Woomera tracking stations. The data rate is one observation per ten minutes for each station during visibility periods. Tracking was simulated for a period of 30 days (2,592,000 seconds).

Comparison runs were made with the same tracking data making identical contributions for both methods. The A^TA matrices were calculated with the

A matrix defined for the respective systems. The parabolic degenerate system is the primitive system without tuning. The following excerpts have been chosen from the comparison runs for reproduction here:

Matrix	Printout Code	Description
A^TA	OMGA 2	Tracking normal matrix
$(A^TA)^{-1}$	OMGA 1	Covariance matrix
Correlation Matrix	Correlation Matrix	Standard deviation on the main diagonal, otherwise correlations
Check Inverse	Inverse of OMGA 1	Check inverse

The rows and columns of the individual matrices are identified by index numbers to be interpreted as follows:

Row and Column Key Number	Cartesian Interpretation	Parabolic Degenerate Interpretation
1	x	α_1
2	y	α_2
3	z	α_3
4	$\dot{x}$	$\alpha_{\bar{1}}$
5	$\dot{y}$	$\alpha_{\bar{2}}$
6	$\dot{z}$	$\alpha_{\bar{3}}$

Specimen matrices are reproduced in Tables 1 through 8 for the following times:

- 5 days 432,000 seconds
- 10 days 864,000 seconds
- 20 days 1,728,000 seconds
- 30 days 2,592,000 seconds

It can be seen from the specimens that by 20 and 30 days, the Cartesian system is developing difficulties, while the parabolic degenerate system is still functioning well.

9. NEW TECHNOLOGY

The method and technique of formulating and using α-variables as presented in this report were developed independently by TRW Systems. The responsible investigator is Mr. W. W. Lemmon.

This technique was used to develop the results presented here, which could be applied to the lunar orbiter. No new technology was developed under the contract covering this report.

REFERENCES

1) L. S. Diamant and W. W. Lemmon, "A Unique System of Analytical State Transition Partial Derivatives," Proceedings of Space Flight Mechanics Specialist Conference," American Astronautical Society, Denver, Colorado; July 6, 1966.

2) J. E. Brooks and W. W. Lemmon, "A Universal Formulation for Conic Trajectories - Basic Variables and Relationships," TRW Systems (formerly TRW Space Technology Laboratories) Report 3400-6019-TU000; February 1965. (This report was prepared under Contract No. NASA9-2938 to document previously unpublished work of W. W. Lemmon)

3) R. H. Battin, Astronautical Guidance, McGraw-Hill Book Co., New York, N. Y.; 1964.

4) W. W. Lemmon, Notes and miscellany.

EPOCH TIME INDEX	COOR SYSTEM INDEX	CENTRAL BODY INDEX	LAST RECYCLE INDEX	CURRENT TIME INDEX
1	10	3		60

T	P(X)	P(Y)	P(Z)	V(X)	V(Y)	V(Z)
.43200000 06	-.81536953 07	-.27735644 07	-.34140698 07	.19955898 04	-.23410076 04	-.27452173 04

MATRIX	MU(1)	RHO(1)	TAU(1)	MU(2)	RHO(2)	TAU(2)
SPR	1	FP	A(K)	1	FP	A(K)

OMGA 2

	1	2	3	4	5	6
1	.14214590 05	.28729902 04	.36400284 04	-.79122571 07	.15679340 08	.18452426 08
2	.28729902 04	.58075718 03	.73580024 03	-.15989626 07	.31690562 07	.37295204 07
3	.36400284 04	.73580024 03	.93224840 03	-.20258670 07	.40151357 07	.47252491 07
4	-.79122571 07	-.15989626 07	-.20258670 07	.44049133 10	-.87275670 10	-.10271211 11
5	.15679340 08	.31690562 07	.40151357 07	-.87275670 10	.17295182 11	.20354019 11
6	.18452426 08	.37295204 07	.47252491 07	-.10271211 11	.20354019 11	.23953887 11

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OMGA 1

	1	2	3	4	5	6
1	.28987098 02	-.52847186 01	-.61122963 02	.28267854-01	-.24632771-01	.23602408-01
2	-.52847186 01	.22447526 03	-.19339577 03	.11660730-01	-.87979150-01	.87028594-01
3	-.61122963 02	-.19339577 03	.38573629 03	-.10375653 00	.15788429 00	-.17754299 00
4	.28267854-01	.11660730-01	-.10375653 00	.42317594-04	-.42528873-04	.51159232-04
5	-.24632771-01	-.87979150-01	.15788429 00	-.42528873-04	.10257965-03	-.10387122-03
6	.23602408-01	.87028594-01	-.17754299 00	.51159232-04	-.10387122-03	.11348903-03

CORRELATION MATRIX

	1	2	3	4	5	6
1	.53839667 01	-.65514170-01	-.57803851 00	.80710488 00	-.45173141 00	.41150679 00
2	-.65514170-01	.14982498 02	-.65723012 00	.11964127 00	-.57978222 00	.54525641 00
3	-.57803851 00	-.65723012 00	.19640170 02	-.81210036 00	.79371220 00	-.84855763 00
4	.80710488 00	.11964127 00	-.81210036 00	.65051975-02	-.64549483 00	.73822134 00
5	-.45173141 00	-.57978222 00	.79371219 00	-.64549483 00	.10128161-01	-.96269269 00
6	.41150678 00	.54525641 00	-.84855764 00	.73822135 00	-.96269268 00	.10653123-01

INVERSE OF OMGA 1

	1	2	3	4	5	6
1	.13818519 05	.27929370 04	.35386024 04	-.76917939 07	.15242452 08	.17938269 08
2	.27929370 04	.56457701 03	.71530023 03	-.15544031 07	.30807533 07	.36256002 07
3	.35386024 04	.71530023 03	.90627523 03	-.19694108 07	.39032574 07	.45935838 07
4	-.76917939 07	-.15544031 07	-.19694108 07	.42821981 10	-.84843843 10	-.99850186 10
5	.15242452 08	.30807533 07	.39032574 07	-.84843843 10	.16813271 11	.19786877 11
6	.17938269 08	.36256002 07	.45935838 07	-.99850186 10	.19786877 11	.23286439 11

Table 1. Specimen Matrix, 5 Days, Cartesian System

Table 2. Specimen Matrix, 5 Days, Parabolic Degenerate System

EPOCH TIME INDEX	COOR SYSTEM INDEX	CENTRAL BODY INDEX	LAST RECYCLE INDEX	CURRENT TIME INDEX
1	6	3		60

T	P(X)	P(Y)	P(Z)	V(X)	V(Y)	V(Z)
.43200000 06	-.81536953 07	-.27735644 07	-.34140698 07	.19955898 04	-.23410076 04	-.27452173 04

MATRIX	MU(1)	RHO(1)	TAU(1)	MU(2)	RHO(2)	TAU(2)
SPR	1	FP	A(K)	1	FP	A(K)

OMGA 2

	1	2	3	4	5	6
1	.15358176 07	.21274692 06	-.88026310 02	-.45673269 05	-.11229816 02	.15490521 06
2	.21274692 06	.45198797 07	-.88921397 01	-.65377570 04	.18388717 03	.59091467 05
3	-.88026310 02	-.88921397 01	.11844055-01	.34922958 01	-.68421036-03	-.21297107 01
4	-.45673269 05	-.65377570 04	.34922958 01	.18692525 04	.17716452 00	-.42087692 04
5	-.11229816 02	.18388717 03	-.68421036-03	.17716452 00	.19515007-01	.16511509 01
6	.15490521 06	.59091467 05	-.21297107 01	-.42087692 04	.16511509 01	.28594721 05

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OMGA 1

	1	2	3	4	5	6
1	.62738246-05	-.17991912-06	.35364077-01	.22249163-04	.87303789-02	-.28210617-04
2	-.17991912-06	.37413195-06	-.15697736-02	.13099858-05	-.37375888-02	.49323755-06
3	.35364077-01	-.15697736-02	.48900902 03	-.62038385 00	.78883340 02	-.24777904 00
4	.22249163-04	.13099858-05	-.62038385 00	.28058000-02	-.67699900-01	.24744389-03
5	.87303789-02	-.37375888-02	.78883340 02	-.67699900-01	.99043516 02	-.49379460-01
6	-.28210617-04	.49323755-06	-.24777904 00	.24744389-03	-.49379460-01	.20759408-03

CORRELATION MATRIX

	1	2	3	4	5	6
1	.25047604-02	-.11743532 00	.63846588 00	.16769462 00	.35023042 00	-.78169782 00
2	-.11743532 00	.61166327-03	-.11605562 00	.40432055-01	-.61399677 00	.55967543-01
3	.63846588 00	-.11605562 00	.22113549 02	-.52963135 00	.35843781 00	-.77767587 00
4	.16769462 00	.40432055-01	-.52963134 00	.52969803-01	-.12842413 00	.32422081 00
5	.35023042 00	-.61399677 00	.35843781 00	-.12842413 00	.99520609 01	-.34437040 00
6	-.78169782 00	.55967543-01	-.77767587 00	.32422081 00	-.34437040 00	.14408126-01

INVERSE OF OMGA 1

	1	2	3	4	5	6
1	.15358176 07	.21274692 06	-.88026312 02	-.45673269 05	-.11229815 02	.15490520 06
2	.21274692 06	.45198798 07	-.88921410 01	-.65377573 04	.18388717 03	.59091467 05
3	-.88026312 02	-.88921410 01	.11844055-01	.34922958 01	-.68421047-03	-.21297108 01
4	-.45673269 05	-.65377573 04	.34922958 01	.18692525 04	.17716448 00	-.42087692 04
5	-.11229815 02	.18388717 03	-.68421047-03	.17716448 00	.19515007-01	.16511509 01
6	.15490520 06	.59091467 05	-.21297108 01	-.42087692 04	.16511509 01	.28594721 05

EPOCH TIME INDEX	COOR SYSTEM INDEX	CENTRAL BODY INDEX	LAST RECYCLE INDEX	CURRENT TIME INDEX
1	10	3		70

T	P(X)	P(Y)	P(Z)	V(X)	V(Y)	V(Z)
.86400000 06	.55866467 07	.34382809 07	.41628233 07	-.33060661 04	.23727476 04	.27627181 04

MATRIX	MU(1)	RHO(1)	TAU(1)	MU(2)	RHO(2)	TAU(2)
SPR	1	FP	A(K)	1	FP	A(K)

OMGA 2

	1	2	3	4	5	6
1	.12428889 06	.25080092 05	.31702528 05	-.69546467 08	.13692207 09	.16128760 09
2	.25080092 05	.50610953 04	.63974795 04	-.14033175 08	.27629542 08	.32546233 08
3	.31702528 05	.63974795 04	.80867628 04	-.17738623 08	.34925179 08	.41140133 08
4	-.69546467 08	-.14033175 08	-.17738623 08	.38916527 11	-.76614978 11	-.90248896 11
5	.13692207 09	.27629542 08	.34925179 08	-.76614978 11	.15083987 12	.17768202 12
6	.16128760 09	.32546233 08	.41140133 08	-.90248896 11	.17768202 12	.20930086 12

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OMGA 1

	1	2	3	4	5	6
1	.97801261 01	-.18121586 01	-.21166098 02	.11050519-01	-.62938592-02	.70135625-02
2	-.18121586 01	.63757499 02	-.44152804 02	-.16118678-02	-.92626848-03	.25215611-03
3	-.21166098 02	-.44152804 02	.12405335 03	-.39271548-01	.17062479-01	-.32625973-01
4	.11050519-01	-.16118678-02	-.39271548-01	.18470016-04	-.86935169-05	.14798627-04
5	-.62938592-02	-.92626848-03	.17062479-01	-.86935169-05	.18621582-04	-.17916721-04
6	.70135625-02	.25215611-03	-.32625973-01	.14798627-04	-.17916721-04	.22560200-04

CORRELATION MATRIX

	1	2	3	4	5	6
1	.31273193 01	-.72570200-01	-.60766513 00	.82219873 00	-.46637634 00	.47216628 00
2	-.72570200-01	.79848293 01	-.49646471 00	-.46971037-01	-.26882085-01	.66486332-02
3	-.60766513 00	-.49646471 00	.11137924 02	-.82042734 00	.35500101 00	-.61671958 00
4	.82219873 00	-.46971037-01	-.82042735 00	.42975756-02	-.46876327 00	.72496378 00
5	-.46637634 00	-.26882085-01	.35500101 00	-.46876327 00	.43152731-02	-.87413552 00
6	.47216628 00	.66486332-02	-.61671958 00	.72496378 00	-.87413552 00	.47497579-02

INVERSE OF OMGA 1

	1	2	3	4	5	6
1	.13063127 06	.26359917 05	.33320294 05	-.73095363 08	.14390913 09	.16951802 09
2	.26359917 05	.53193511 04	.67239282 04	-.14749305 08	.29039458 08	.34207046 08
3	.33320294 05	.67239282 04	.84994108 04	-.18643850 08	.36707386 08	.43239487 08
4	-.73095363 08	-.14749305 08	-.18643850 08	.40902323 11	-.80524608 11	-.94854251 11
5	.14390913 09	.29039458 08	.36707386 08	-.80524608 11	.15853714 12	.18674902 12
6	.16951802 09	.34207046 08	.43239487 08	-.94854251 11	.18674902 12	.21998136 12

Table 3. Specimen Matrix, 10 Days, Cartesian System

Table 4. Specimen Matrix, 10 Days, Parabolic Degenerate System

EPOCH TIME INDEX	COOR SYSTEM INDEX	CENTRAL BODY INDEX	LAST RECYCLE INDEX	CURRENT TIME INDEX
1	6	3		70

T	P(X)	P(Y)	P(Z)	V(X)	V(Y)	V(Z)
.86400000 06	.55866467 07	.34382809 07	.41628233 07	-.33060661 04	.23727476 04	.27627181 04

MATRIX	MU(1)	RHO(1)	TAU(1)	MU(2)	RHO(2)	TAU(2)
SPR	1	FP	A(K)	1	FP	A(K)

OMGA 2

	1	2	3	4	5	6
1	.30909257 07	.39025904 06	-.19177948 03	-.19286995 06	.10163454 01	.11902940 06
2	.39025904 06	.63057469 07	-.10530671 02	-.22444592 05	.66413257 02	.59224549 05
3	-.19177948 03	-.10530671 02	.20724703-01	.12574432 02	-.51485901-02	-.18717063 01
4	-.19286995 06	-.22444592 05	.12574432 02	.16328993 05	-.10650351 01	-.22129991 03
5	.10163454 01	.66413257 02	-.51485901-02	-.10650351 01	.56931194-01	.57619346 01
6	.11902940 06	.59224549 05	-.18717063 01	-.22129991 03	.57619346 01	[illegible]

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OMGA 1

	1	2	3	4	5	6
1	.28724225-05	-.24695879-07	.11021172-01	.25451678-04	.21718471-02	-.71286229-05
2	-.24695879-07	.16350682-06	-.29602485-03	.14610145-06	-.20084611-03	-.13329202-06
3	.11021172-01	-.29602485-03	.13619660 03	.25563053-01	.15422961 02	-.24495743-01
4	.25451678-04	.14610145-06	.25563053-01	.34245181-03	.14717168-01	-.65446744-04
5	.21718471-02	-.20084611-03	.15422961 02	.14717168-01	.20160928 02	-.72151872-02
6	-.71286229-05	-.13329202-06	-.24495743-01	-.65446744-04	-.72151872-02	.40098528-06

CORRELATION MATRIX

	1	2	3	4	5	6
1	.16948223-02	-.36035652-01	.55721201 00	.81150744 00	.28539726 00	-.66422810 00
2	-.36035652-01	.40435977-03	-.62730258-01	.19524811-01	-.11062167 00	-.52056143-01
3	.55721201 00	-.62730258-01	.11670330 02	.11836678 00	.29432648 00	-.33146926 00
4	.81150744 00	.19524812-01	.11836678 00	.18505454-01	.17712067 00	-.55850116 00
5	.28539726 00	-.11062167 00	.29432648 00	.17712067 00	.44900923 01	-.25376289 00
6	-.66422810 00	-.52056143-01	-.33146926 00	-.55850116 00	-.25376289 00	.63323399-02

INVERSE OF OMGA 1

	1	2	3	4	5	6
1	.30909258 07	.39025905 06	-.19177949 03	-.19286996 06	.10163471 01	.11902941 06
2	.39025905 06	.63057469 07	-.10530672 02	-.22444592 05	.66413258 02	.59224551 05
3	-.19177949 03	-.10530672 02	.20724704-01	.12574433 02	-.51485902-02	-.18717065 01
4	-.19286996 06	-.22444592 05	.12574433 02	.16328993 05	-.10650352 01	-.22130009 03
5	.10163471 01	.66413258 02	-.51485902-02	-.10650352 01	.56931194-01	.57619348 01
6	.11902941 06	.59224551 05	-.18717065 01	-.22130009 03	.57619348 01	.45828394 05

EPOCH TIME INDEX	COOR SYSTEM INDEX	CENTRAL BODY INDEX	LAST RECYCLE INDEX	CURRENT TIME INDEX
1	10	3		85

T	P(X)	P(Y)	P(Z)	V(X)	V(Y)	V(Z)
.17280000 07	.27933244 07	.47667849 07	.56951947 07	-.43988774 04	.13083418 04	.14837945 04

MATRIX	MU(1)	RHO(1)	TAU(1)	MU(2)	RHO(2)	TAU(2)
SPR	1	FP	A(K)	1	FP	A(K)

CMGA 2

	1	2	3	4	5	6
1	.10562611 07	.21289690 06	.26883606 06	-.59266553 09	.11632717 10	.13700816 10
2	.21289690 06	.42911245 05	.54186264 05	-.11945504 09	.23446600 09	.27615004 09
3	.26883606 06	.54186264 05	.68423884 05	-.15084204 09	.29607248 09	.34870913 09
4	-.59266553 09	-.11945504 09	-.15084204 09	.33254584 12	-.65270834 12	-.76874884 12
5	.11632717 10	.23446600 09	.29607248 09	-.65270834 12	.12811247 13	.15088869 13
6	.13700816 10	.27615004 09	.34870913 09	-.76874884 12	.15088869 13	.17771413 13

LMDAPR

OMGA 1

	1	2	3	4	5	6
1	.10366706 02	.13011772 02	-.41657357 02	.14411776-01	-.34790504-01	.33933057-01
2	.13011772 02	.60761819 02	-.88861703 02	.19975779-01	-.77455512-01	.72368068-01
3	-.41657357 02	-.88861703 02	.21385666 03	-.65185908-01	.17750900 00	-.17495132 00
4	.14411776-01	.19975779-01	-.65185908-01	.22409788-04	-.53694942-04	.53859754-04
5	-.34790504-01	-.77455512-01	.17750900 00	-.53694942-04	.18159978-03	-.17338818-03
6	.33933057-01	.72368068-01	-.17495132 00	.53859754-04	-.17338818-03	.16743711-03

CORRELATION MATRIX

	1	2	3	4	5	6
1	.32197370 01	.51844259 00	-.88472830 00	.94553634 00	-.80183063 00	.81447308 00
2	.51844259 00	.77949868 01	-.77953900 00	.54133917 00	-.73735949 00	.71747327 00
3	-.88472830 00	-.77953900 00	.14623839 02	-.94161516 00	.90074386 00	-.92454908 00
4	.94553634 00	.54133917 00	-.94161516 00	.47338978-02	-.84169903 00	.87926466 00
5	-.80183064 00	-.73735949 00	.90074385 00	-.84169903 00	.13475896-01	-.99434239 00
6	.81447308 00	.71747328 00	-.92454908 00	.87926467 00	-.99434239 00	.12939749-01

INVERSE OF OMGA 1

	1	2	3	4	5	6
1	.30482743 06	.61439885 05	.77583347 05	-.17103834 09	.33570903 09	.39539238 09
2	.61439885 05	.12383964 05	.15637867 05	-.34472964 08	.67664630 08	.79694229 08
3	.77583347 05	.15637867 05	.19746801 05	-.43530722 08	.85443647 08	.10063408 09
4	-.17103834 09	-.34472964 08	-.43530722 08	.95972099 11	-.18836543 12	-.22185366 12
5	.33570903 09	.67664630 08	.85443647 08	-.18836543 12	.36972038 12	.43545034 12
6	.39539238 09	.79694229 08	.10063408 09	-.22185366 12	.43545034 12	.51286602 12

Table 5. Specimen Matrix, 20 Days, Cartesian System

Table 6. Specimen Matrix, 20 Days, Parabolic Degenerate System

EPOCH TIME INDEX	COOR SYSTEM INDEX	CENTRAL BODY INDEX	LAST RECYCLE INDEX	CURRENT TIME INDEX
1	6	3		85

T	P(X)	P(Y)	P(Z)	V(X)	V(Y)	V(Z)
.17280000 07	.27933244 07	.47667849 07	.56951947 07	-.43988774 04	.13083418 04	.14837945 04

MATRIX	MU(1)	RHO(1)	TAU(1)	MU(2)	RHO(2)	TAU(2)
SPR	1	FP	A(K)	1	FP	A(K)

OMGA 2

	1	2	3	4	5	6
1	.63501468 07	.31521663 06	-.30154935 03	-.81232181 06	.28528233 02	.31964844 06
2	.31521663 06	.17265242 08	-.85564040 02	-.40814821 04	.11436806 03	.29047923 05
3	-.30154935 03	-.85564040 02	.37525501-01	.37743036 02	-.10962498-01	-.26697095 01
4	-.81232181 06	-.40814821 04	.37743036 02	.13869196 06	-.60045397 01	-.41182047 05
5	.28528233 02	.11436806 03	-.10962498-01	-.60045397 01	.94129454-01	.37669124 01
6	.31964844 06	.29047923 05	-.26697095 01	-.41182047 05	.37669124 01	.84438013 05

LMDAPR OMGA 1

	1	2	3	4	5	6
1	.84668820-06	.80430083-10	.30378113-02	.37662716-05	.38894399-03	-.12896700-05
2	.80430083-10	.59215104-07	.17880559-03	-.61401680-07	-.53359683-04	-.42588304-07
3	.30378113-02	.17880559-03	.51009945 02	.13101226-02	.52683204 01	-.95447016-02
4	.37662716-05	-.61401680-07	.13101226-02	.28911045-04	.93541918-03	-.13630413-06
5	.38894399-03	-.53359683-04	.52683204 01	.93541918-03	.11297284 02	-.13352265-02
6	-.12896700-05	-.42588304-07	-.95447016-02	-.13630413-06	-.13352265-02	.16431142-04

CORRELATION MATRIX

	1	2	3	4	5	6
1	.92015662-03	.35920346-03	.46224434 00	.76123365 00	.12575877 00	-.34576655 00
2	.35920345-03	.24334154-03	.10288153 00	-.46928006-01	-.65239401-01	-.43175782-01
3	.46224434 00	.10288153 00	.71421247 01	.34115569-01	.21946119 00	-.32968641 00
4	.76123365 00	-.46928006-01	.34115569-01	.53768992-02	.51759175-01	-.62537891-02
5	.12575877 00	-.65239401-01	.21946119 00	.51759175-01	.33611432 01	-.98001797-01
6	-.34576655 00	-.43175783-01	-.32968641 00	-.62537891-02	-.98001796-01	.40535345-02

INVERSE OF OMGA 1

	1	2	3	4	5	6
1	.63501471 07	.31521664 06	-.30154937 03	-.81232186 06	.28528235 02	.31964846 06
2	.31521664 06	.17265242 08	-.85564041 02	-.40814841 04	.11436806 03	.29047923 05
3	-.30154937 03	-.85564041 02	.37525502-01	.37743039 02	-.10962498-01	-.26697101 01
4	-.81232186 06	-.40814841 04	.37743039 02	.13869196 06	-.60045399 01	-.41182049 05
5	.28528235 02	.11436806 03	-.10962498-01	-.60045399 01	.94129454-01	.37669125 01
6	.31964846 06	.29047923 05	-.26697101 01	-.41182049 05	.37669125 01	.84438013 05

Table 7. Specimen Matrix, 30 Days, Cartesian System

EPOCH TIME INDEX	COOR SYSTEM INDEX	CENTRAL BODY INDEX	LAST RECYCLE INDEX	CURRENT TIME INDEX
1	10	3		95

T	P(X)	P(Y)	P(Z)	V(X)	V(Y)	V(Z)
.25920000 07	-.49036470 06	.52715884 07	.62433182 07	-.46810420 04	.10656331 03	.54435028 02

MATRIX	MU(1)	RHO(1)	TAU(1)	MU(2)	RHO(2)	TAU(2)
SPR	1	FP	A(K)	1	FP	A(K)

OMGA 2

	1	2	3	4	5	6
1	.23672964 07	.47694432 06	.60213610 06	-.13292678 10	.26067067 10	.30703160 10
2	.47694432 06	.96091481 05	.12131426 06	-.26780933 09	.52517938 09	.61858382 09
3	.60213610 06	.12131426 06	.15315777 06	-.33810572 09	.66303225 09	.78095420 09
4	-.13292678 10	-.26780933 09	-.33810572 09	.74640464 12	-.14636990 13	-.17240215 13
5	.26067067 10	.52517938 09	.66303225 09	-.14636990 13	.28783307 13	.33808262 13
6	.30703160 10	.61858382 09	.78095420 09	-.17240215 13	.33808262 13	.39821147 13

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OMGA 1

	1	2	3	4	5	6
1	.25301046 01	.10278217 01	-.61458337 01	.26076636-02	-.33194908-02	.30420745-02
2	.10278217 01	.32281060 02	-.22120508 02	.11691767-03	-.22408809-01	.17606897-01
3	-.61458337 01	-.22120508 02	.37734768 02	-.92408155-02	.18613470-01	-.19029175-01
4	.26076636-02	.11691767-03	-.92408155-02	.40637546-05	-.21419583-05	.33614220-05
5	-.33194908-02	-.22408809-01	.18613470-01	-.21419583-05	.52101791-04	-.42771881-04
6	.30420745-02	.17606897-01	-.19029175-01	.33614220-05	-.42771881-04	.36420078-04

CORRELATION MATRIX

	1	2	3	4	5	6
1	.15906303 01	.11372990 00	-.62898561 00	.81323971 00	-.28911850 00	.31690580 00
2	.11372990 00	.56816424 01	-.63379724 00	.10208043-01	-.54640988 00	.51349771 00
3	-.62898561 00	-.63379724 00	.61428632 01	-.74623516 00	.41978813 00	-.51330879 00
4	.81323972 00	.10208043-01	-.74623516 00	.20158756-02	-.14720445 00	.27630509 00
5	-.28911850 00	-.54640988 00	.41978813 00	-.14720445 00	.72181570-02	-.98188723 00
6	.31690580 00	.51349771 00	-.51330879 00	.27630509 00	-.98188724 00	.60349045-02

INVERSE OF OMGA 1

	1	2	3	4	5	6
1	.82189873 06	.16558926 06	.20905431 06	-.46150767 09	.90501820 09	.10659779 10
2	.16558926 06	.33362017 05	.42119111 05	-.92979572 08	.18233604 09	.21476493 09
3	.20905431 06	.42119111 05	.53174905 05	-.11738537 09	.23019690 09	.27113799 09
4	-.46150767 09	-.92979572 08	-.11738537 09	.25914644 12	-.50817956 12	-.59856061 12
5	.90501820 09	.18233604 09	.23019690 09	-.50817956 12	.99654528 12	.11737833 13
6	.10659779 10	.21476493 09	.27113799 09	-.59856061 12	.11737833 13	.13825437 13

Table 8. Specimen Matrix, 30 Days, Parabolic Degenerate System

EPOCH TIME INDEX	COOR SYSTEM INDEX	CENTRAL BODY INDEX	LAST RECYCLE INDEX	CURRENT TIME INDEX
1	6	3		95

T	P(X)	P(Y)	P(Z)	V(X)	V(Y)	V(Z)
.25920000 07	-.49036470 05	.52715884 07	.62433182 07	-.46810420 04	.10656331 03	.54435028 02

MATRIX	MU(1)	RHO(1)	TAU(1)	MU(2)	RHO(2)	TAU(2)
SPR	1	FP	A(K)	1	FP	A(K)

OMGA 2

$\bar{A}^T A$

	1	2	3	4	5	6
1	.82622341 07	.18521611 06	-.35238927 03	-.13839021 07	.98854928 01	.32835775 06
2	.18521611 06	.22486780 08	-.72621117 02	.31783878 05	-.56635145 02	.45771943 05
3	-.35238927 03	-.72621117 02	.42168264-01	.51415062 02	-.12120498-01	-.20565200 01
4	-.13839021 07	.31783878 05	.51415062 02	.31079926 06	.13720937-01	-.45639791 05
5	.98854928 01	-.56635145 02	-.12120498-01	.13720937-01	.12039219 00	.77042235 00
6	.32835775 06	.45771943 05	-.20565200 01	-.45639791 05	.77042235 00	[illegible] 05

LMDAPR

OMGA 1

	1	2	3	4	5	6
1	.71814132-06	.30084985-08	.29248810-02	.25368927-05	.24431816-03	[illegible]
2	.30084985-08	.45071905-07	.13411033-[illegible]	-.1[illegible]097-07	.34706736-04	-.38629560-07
3	.29248810-02	.13411033-03	.43403768 02	.48083618-02	.42365570 01	-.69541445-02
4	.25368927-05	-.19073097-07	.48083618-02	.13393893-04	.27949200-03	-.22212962-05
5	.24431816-03	.34706736-04	.42365570 01	.27949200-03	.87334518 01	[illegible]
6	-.12027008-05	-.38629560-07	-.69541445-02	-.22212962-05	-.70554[illegible]-03	[illegible]

CORRELATION MATRIX

	1	2	3	4	5	6
1	.84743228-03	.16722141-01	.52388929 00	.81798196 00	.97556861-01	-.38654131 00
2	.16722141-01	.21230145-03	.95883866-01	-.24547928-01	.55318177-01	-.49557579-01
3	.52388929 00	.95883866-01	.65881536 01	.19942519 00	.21759874 00	-.28749035 00
4	.81798196 00	-.24547928-01	.19942519 00	.36597668-02	.25841816-01	-.16530894 00
5	.97556861-01	.55318177-01	.21759874 00	.25841816-01	.29552413 01	-.65023791-01
6	-.38654131 00	-.49557579-01	-.28749035 00	-.16530894 00	-.65023791-01	.36716114-02

INVERSE OF OMGA 1

	1	2	3	4	5	6
1	.82622336 07	.18521610 06	-.35238925 03	-.13839020 07	.98854936 01	.32835773 06
2	.18521610 06	.22486780 08	-.72621116 02	.31783879 05	-.56635145 02	.45771943 05
3	-.35238925 03	-.72621116 02	.42168263-01	.51415060 02	-.12120498-01	-.20565191 01
4	-.13839020 07	.31783879 05	.51415060 02	.31079924 06	.13720772-01	-.45639788 05
5	.98854936 01	-.56635145 02	-.12120498-01	.13720772-01	.12039219 00	.77042231 00
6	.32835773 06	.45771943 05	-.20565191 01	-.45639788 05	.77042231 00	.95065100 05